Cosmology in a brane-universe

David Langlois
GReCO, Institut d'Astrophysique de Paris (CNRS)
98 bis, Boulevard Arago, 75014 Paris, France

October 15, 2002

Abstract. This contribution presents the cosmological models with *extra dimensions* that have been recently elaborated, which assume that ordinary matter is confined on a surface, called *brane*, embedded in a higher dimensional spacetime.

Keywords: cosmology, extra-dimensions, branes

1. Introduction

The purpose of this contribution is to present a new approach to extra-dimensions, the "braneworld" scenario, where ordinary matter is trapped in a three-dimensional surface, called 'brane', embedded in a higher dimensional space.

This idea must be contrasted with the traditional Kaluza-Klein treatment of extra dimensions, where matter fields live *everywhere* in *compact* extra dimensions and can be described, via a Fourier expansion, as an infinite collection of four-dimensional fields. These so-called Kaluza-Klein modes can be excited only when the energy at disposal exceeds their mass, typically inversely proportional to the size, say R, of the extra-dimensions. Presently, the energy scale reached in colliders is $E_{max} \sim 1$ TeV, which implies that

$$R \lesssim (1\text{TeV})^{-1} \tag{1}$$

in the Kaluza-Klein picture.

In braneworlds, if only gravity propagates in the higher-dimensional spacetime, called *bulk*, the size of the extra-dimensions can be much larger than previously believed since four-dimensional gravity is only tested on scales above about a millimeter. Moreover, the four-dimensional Planck mass M_p is in this context only a "projection" of the higher-dimensional (fundamental) Planck mass, which can be lower than M_p, thus offering a new perspective on the hierarchy problem and suggesting the possibility that quantum gravity might be closer than previously thought.

This new approach to dimensional reduction has been motivated by the latest developments in string/M-theory, in particular the emphasis on branes as loci where open strings end and define gauge fields, as well as the Horava-Witten model where gauge fields are defined on hyperplanes located at fixed points of the Z_2 orbifold symmetric eleventh dimension (Horava and Witten, 1996).

The cosmological consequences of these models have been studied in various ways. The approaches tend to differ if one is a string theorist or a cosmologist. The former prefers to work with models derived from string theory but often too complex to tackle realistic cosmology whereas the latter sacrifices some aspects of the high energy phenomenology in order to get a tractable model. So far, the task is too difficult to be satisfying from both viewpoints, but the hope is that one can learn from these two directions. Here, I focus on the cosmologist's strategy and present string-inspired, rather than string-derived, models, which consist of a self-gravitating brane-universe embedded in a five-dimensional bulk spacetime.

2. The modified Friedmann equation

The main motivation for exploring cosmology in models with extra-dimensions is that specific signatures might be accessible *only* at very high energies, i.e. in the very early universe. One would thus like to investigate what kind of relic imprints could be left and tested today via cosmological observations. This present contribution is devoted to homogeneous brane cosmology and the reader is invited to refer to Nathalie Deruelle's contribution for a review on the important subject of cosmological perturbations in brane cosmology.

In this section, I describe the cosmology of a *self-gravitating brane-universe* embedded in an *empty* five-dimensional spacetime (Binétruy et al., 2000a; Binétruy et al., 2000b; Flanagan et al., 2000; Shiromizu et al., 2000). Assuming isotropy and homogeneity along three of its spatial dimensions (which correspond in the brane to our ordinary spatial dimensions) it is always possible to write the spacetime metric (at least locally in the vicinity of the brane) in the form

$$ds^2 = -n(t,y)^2 dt^2 + a(t,y)^2 \gamma_{ij} dx^i dx^j + dy^2. \qquad (2)$$

where γ_{ij} is the maximally symmetric three-dimensional metric, with either negative, vanishing or positive spatial curvature (respectively labelled by $k = -1$, 0 or 1).

In these coordinates, our brane-universe is always located at $y = 0$, and the cosmological scale factor for a brane observer is $a_0(t) \equiv a(t, 0)$. It is always possible to rescale the time coordinate so that it corresponds on the brane to the usual cosmic time, i.e. $n_0(t) \equiv n(t, 0) = 1$.

The total energy-momentum tensor can be decomposed into a bulk energy-momentum tensor, which will be assumed to vanish here, and a brane energy-momentum tensor, the latter being of the form

$$T^A_B = S^A_B \delta(y) = \{\rho_b, P_b, P_b, P_b, 0\}\delta(y), \tag{3}$$

where the delta function expresses the localisation of matter at the brane position $y = 0$. The quantities ρ_b and P_b are respectively the total energy density and pressure in the brane and depend only on time. Allowing for a cosmological constant in the bulk, Λ, the five-dimensional Einstein equations read

$$G_{AB} + \Lambda g_{AB} = \kappa^2 T_{AB}, \tag{4}$$

where κ^2 is the gravitational coupling (and scales like the inverse of the cube of the fundamental mass in five dimensions).

Instead of solving directly Einstein's equations with a distributional matter source, one can first obtain the general solution in the bulk and then apply boundary conditions at the brane location. The latter can be obtained from the integration of Einstein's equations in the vicinity of the brane. According to these junction conditions, the metric must be continuous and the jump of the extrinsic curvature tensor K_{AB} (related to the derivatives of the metric with respect to y) depends on the distributional energy-momentum tensor,

$$\left[K^A_B - K\delta^A_B\right] = \kappa^2 S^A_B, \tag{5}$$

where the brackets denote the jump at the brane and the extrinsic curvature tensor is defined by $K_{AB} = h^C_A \nabla_C n_B$, n^A being the unit vector normal to the brane and $h_{AB} = g_{AB} - n_A n_B$ the induced metric.

Assuming moreover that the brane is mirror symmetric, like in the Horava-Witten model, the jump in the extrinsic curvature is twice its value on one side. Substituting the ansatz metric (2) in (5), one ends up with the two junction conditions:

$$\left(\frac{n'}{n}\right)_{0^+} = \frac{\kappa^2}{6}\left(3p_b + 2\rho_b\right), \qquad \left(\frac{a'}{a}\right)_{0^+} = -\frac{\kappa^2}{6}\rho_b. \tag{6}$$

One can then solve explicitly (Binétruy et al., 2000b) Einstein's equations (4) for the metric ansatz (2). One finds in particular that the geometry induced *in the brane* is governed by the equation

$$H_0^2 \equiv \frac{\dot{a}_0^2}{a_0^2} = \frac{\kappa^4}{36}\rho_b^2 + \frac{\Lambda}{6} - \frac{k}{a_0^2} + \frac{\mathcal{C}}{a_0^4}. \tag{7}$$

where $\mathcal{C}$ is an integration constant. This equation is analogous to the (first) Friedmann equation, since it relates the Hubble parameter to the energy density, but is nevertheless different [the usual Friedmann equation reads $H^2 = (8\pi G/3)\rho$]. A striking property of this equation is that the energy density of the brane enters quadratically on the right hand side in contrast with the standard four-dimensional Friedmann equation and its linear dependence on the energy density. Another consequence of the five-dimensional Einstein equations (4) is that the energy conservation equation is unchanged and still reads

$$\dot{\rho}_b + 3H(\rho_b + p_b) = 0. \tag{8}$$

In the simplest case where $\Lambda = 0$ and $\mathcal{C} = 0$, one can easily solve the above cosmological equations (7-8) for a perfect fluid with an equation of state $p_b = w\rho_b$ (w constant). One finds that the evolution of the scale factor is given by

$$a_0(t) \propto t^{\frac{1}{3(1+w)}}. \tag{9}$$

In the most interesting cases for cosmology, radiation and pressureless matter, one finds respectively $a \sim t^{1/4}$ (instead of the standard $a \sim t^{1/2}$) and $a \sim t^{1/3}$ (instead of $a \sim t^{2/3}$). Such behaviour is problematic because it would spoil nucleosynthesis. Indeed, the nucleosynthesis scenario depends on the balance between the microphysical reaction rates and the expansion rate of the universe, and changing the latter in a drastic way between nucleosynthesis and today alters dramatically the predictions for the light element abundances.

The modified Friedmann law (7), with the ρ_b^2 term but without the bulk cosmological constant (and without the $\mathcal{C}$ term) was derived just before a new model describing a flat (Minkowski) world with one extra-dimension was proposed by Randall and Sundrum (Randall and Sundrum, 1999). The new ingredient of this model was to endow our brane-world with a tension (constant energy density) and the five-dimensional bulk with a *negative* cosmological constant, the two being fine-tuned so that the effective four-dimensional Hubble parameter is zero in (7) (taking $\mathcal{C} = 0$). It turns out that such a set-up gives the usual four-dimensional gravity, except on very small scales (Randall and Sundrum, 1999; Garriga and Tanaka, 2000).

The recovery of ordinary gravity suggested that the cosmological generalization of the Randall-Sundrum model should be compatible with standard cosmology at small energy scales, as this was quickly verified (Csaki et al., 1999; Cline et al., 1999). Let us see how it works. In order to go beyond the Minkowski geometry and consider a non trivial cosmology in the brane, one must assume that the total energy density in the brane, ρ_b, consists of two parts,

$$\rho_b = \sigma + \rho, \tag{10}$$

the tension σ, constant in time, and the usual cosmological energy density ρ. Substituting this decomposition into (7), one obtains

$$H^2 = \left(\frac{\kappa^4}{36}\sigma^2 + \frac{\Lambda}{6}\right) + \frac{\kappa^4}{18}\sigma\rho + \frac{\kappa^4}{36}\rho^2 - \frac{k}{a^2} + \frac{\mathcal{C}}{a^4}. \tag{11}$$

Let us now fine-tune the brane tension and the bulk cosmological constant so that the term between parentheses vanishes (or at least is extremely small). For $\rho \ll \sigma$, the next term dominates over the ρ^2 and *one thus recovers the usual Friedmann equation at low energy*, with the identification

$$8\pi G = \frac{\kappa^4}{6}\sigma, \tag{12}$$

which also agrees with Newton's constant deduced from gravitational interaction between test masses. The third term on the right hand side of (11), quadratic in the energy density, provides a *high-energy correction* to the Friedmann equation which becomes significant when the value of the energy density approaches the value of the tension σ and dominates at higher energy densities. In the very high energy regime, $\rho \gg \sigma$, one thus recovers the unconventional behaviour of (9) since the bulk cosmological constant becomes negligible. For an equation of state $p = w\rho$, with w constant, the conservation equation (8) gives as usual

$$\rho = \rho_* a^{-q}, \quad q \equiv 3(w+1), \tag{13}$$

which after substitution in the Friedmann equation (11) yields (for $k = 0$ and $\mathcal{C} = 0$)

$$a(t) = \left[q m_* t \left(1 + \frac{q}{2}\mu t\right)\right]^{1/q}, \tag{14}$$

where we have introduced the two mass scales

$$m_* \equiv \frac{\kappa^2}{6}\rho_*, \qquad \mu \equiv \sqrt{-\Lambda/6}. \tag{15}$$

One sees that the evolution of the scale factor interpolates between the low energy regime and the high energy regime and that the transition time is of the order of μ^{-1}, which is the characteristic scale associated with the cosmological constant.

Finally, the last term on the right hand side of (11) behaves like radiation and arises from the integration constant $\mathcal{C}$. This constant $\mathcal{C}$ is analogous to the Schwarzschild mass, as we will see in the next section, and it is related to the bulk Weyl tensor, which vanishes when $\mathcal{C} = 0$.

In a cosmological context, this term is constrained to be small enough at the time of nucleosynthesis in order to satisfy the constraints on the number of extra light degrees of freedom.

The metric outside the brane can be also determined explicitly (Binétruy et al., 2000b). In the special case $\mathcal{C} = 0$, the metric has a very simple form and its components are given by

$$a(t,y) = a_0(t)\left(\cosh \mu y - \eta \sinh \mu|y|\right) \tag{16}$$

$$n(t,y) = \cosh \mu y - \tilde{\eta} \sinh \mu|y| \tag{17}$$

where

$$\eta = 1 + \frac{\rho}{\sigma}, \qquad \tilde{\eta} = \eta + \frac{\dot{\eta}}{H_0}. \tag{18}$$

The Randall-Sundrum model corresponds to $\rho = 0$, i.e. $\rho_b = \sigma$, which implies $\eta = \tilde{\eta} = 1$ and $a(t,y) = a_0 \exp(-\mu|y|)$.

As explained above, the Randall-Sundrum version of brane cosmology gives at sufficiently late times a cosmological evolution identical to the usual one. The model is thus viable if the low-energy regime encompasses the periods that are well constrained by cosmological observations. This essentially means that nucleosynthesis must take place in the low-energy regime. This is the case if the energy scale associated with the tension is higher than the nucleosynthesis energy scale, i.e.

$$\sigma^{1/4} \gtrsim 1 \text{ MeV}. \tag{19}$$

Combining this with (12) implies for the fundamental mass scale (defined by $\kappa^2 = M^{-3}$) the constraint $M \gtrsim 10^4$ GeV. There is however a more stringent constraint: the requirement to recover ordinary gravity down to scales of the submillimeter order, which have been probed by gravity experiments (Hoyle et al., 2001). This implies

$$\ell = \mu^{-1} \lesssim 10^{-1} \text{ mm}, \tag{20}$$

which yields the constraint

$$M \gtrsim 10^8 \text{ GeV}. \tag{21}$$

Another parameter of the model is the Weyl parameter $\mathcal{C}$. As mentioned above, its value is restricted by the bounds on the number of additional relativistic degrees of freedom allowed during nucleosynthesis (usually expressed as the number of additional light neutrino species). Typically, this gives the constraint

$$\frac{\rho_{Weyl}}{\rho_{rad}} \equiv \frac{\mathcal{C}\sigma}{2a^4\mu^2\rho} \lesssim 10\%. \tag{22}$$

3. Moving brane in a static bulk

In the previous section, we considered a specific system of coordinates such that the brane is always at $y = 0$ and the metric is of the form (2). Although this choice is very convenient from the point of view of the brane, it does not give the simplest description of the bulk geometry. Indeed, it turns out that the required 'cosmological symmetries' are so strong that the geometry of the empty bulk is necessarily static, and in an appropriate coordinate system, is described by a metric of the form (Krauss, 1999)

$$ds^2 = -f(R)\,dT^2 + \frac{dR^2}{f(R)} + R^2\,\gamma_{ij}\,dx^i dx^j, \tag{23}$$

where

$$f(R) \equiv k - \frac{\Lambda}{6}R^2 - \frac{\mathcal{C}}{R^2}. \tag{24}$$

The above metric is known as the five-dimensional Schwarzschild-Anti de Sitter (Sch-AdS) metric (with $\Lambda < 0$). It is clear from (24) that $\mathcal{C}$, as noted before, is the five-dimensional analog of the Schwarschild mass (the R^{-2} dependence instead of the usual R^{-1} is simply due to the different dimension of spacetime).

It can be shown explicitly, by considering the appropriate coordinate transformation, that the manifestly static metric (23) indeed coincides with the previous expression (17) for the metric. The simplicity of the metric (23) is however counterbalanced by the fact that *the cosmological brane is necessarily moving* with respect to this coordinate frame.

The trajectory of the brane can be defined by its coordinates $T(\tau)$ and $R(\tau)$ given as functions of a parameter τ. Choosing τ to be the proper time imposes the condition

$$g_{ab}u^a u^b = -f\dot{T}^2 + \frac{\dot{R}^2}{f} = -1, \tag{25}$$

where $u^a = (\dot{T}, \dot{R})$ is the brane velocity and a dot stands for a derivative with respect to the parameter τ. Using this normalization condition (25), one finds that the components of the unit normal vector (defined such that $n_a u^a = 0$ and $n_a n^a = 1$) are given, up to a sign ambiguity, by $n_a = \left(\dot{R}, -\sqrt{f + \dot{R}^2}/f\right)$. The four-dimensional metric induced in the brane worldsheet is then directly given by

$$ds^2 = -d\tau^2 + R(\tau)^2 d\Omega_k^2, \tag{26}$$

which shows clearly that the brane scale factor, denoted a_0 previously, can be identified with the radial coordinate of the brane $R(\tau)$.

The dynamics of the brane is then obtained by writing the junction conditions for the brane. The 'orthogonal' part of the junction conditions yields

$$K^i_j = \frac{\sqrt{f+\dot{R}^2}}{R}\delta^i_j = \frac{\kappa^2}{6}\rho_b, \tag{27}$$

which, after substituting (24) and rearranging, gives exactly the Friedmann equation (7) obtained before. There is also information in the 'longitudinal' part of the junction conditions, which can be rewritten as the standard conservation equation (8). This confirms the complete equivalence between the 'brane-based' and the 'bulk-based' pictures.

Let us add that the metric (23) describes in principle only one side of the brane. In the case of a mirror symmetric brane, as assumed above, the complete spacetime is obtained by gluing, along the brane worldsheet, two copies of a portion of Sch-AdS spacetime. For an asymmetric brane, one can glue two (compatible) portions of different Sch-AdS spacetimes. This can also be generalized to a system of several ('parallel') branes, which can be moving with respect to each other, thus suggesting the possibility of collisions (Langlois et al., 2002a). This idea has recently attracted a lot of attention with the proposal that the cosmological Big Bang might be in fact such a brane collision.

4. Brane radiating gravitons into the bulk

The analysis of the cosmological behaviour of the brane presented in the previous sections was based on the assumption of perfect homogeneity and isotropy along the three ordinary dimensions. In real cosmology, these symmetries hold only on average and there exist fluctuations on small scales, which create gravitational waves that can escape into the bulk. A consequence of this process is that the Weyl parameter $\mathcal{C}$ is no longer constant (Hebecker and March-Russell, 2001).

As I will now summarize, it is possible to build a simplified model that treats *self-consistently* the emission of gravitons, the backreaction on the bulk geometry and the motion, i.e. cosmology, of our brane-universe (Langlois et al., 2002b). This model is based on the simplifying assumption that all gravitons are emitted only in the radial direction, i.e. perpendicularly to the brane. The corresponding bulk energy-momentum tensor is thus of the form

$$T_{AB} = \mathcal{F} k_A k_B, \tag{28}$$

where k^A is an ingoing null vector, which can be normalized so that $k_A u^A = 1$, where u^A is the brane velocity vector. The solution of the bulk Einstein equations is then the generalization to five dimensions of Vaidya's metric, which describes the spacetime surrounding a radiating star. The associated metric reads

$$ds^2 = -f\,(r,\, v)\; dv^2 + 2\, dr\, dv + r^2\, \delta_{ij} dx^i dx^j, \quad f(r,v) = \mu^2 r^2 - \frac{\mathcal{C}(v)}{r^2}. \tag{29}$$

If $\mathcal{C}$ does not depend on v, the above metric is simply a rewriting of the Sch-AdS metric (23) in terms of the null coordinate $v = T + \int dr/f(r)$. Note that, strictly speaking, the Vaidya spacetime corresponds to an *outgoing* radiation flow, whereas in our case, we are interested in an *ingoing* radial flow because the brane, which emits the radiation, is in some sense located at the largest radius of spacetime (when getting away from the brane the radius, or scale factor, decreases).

Einstein's equations relate the energy flux $\mathcal{F}$ to the variation of the Weyl parameter, according to the expression

$$\frac{d\mathcal{C}}{dv} = \frac{2\kappa^2 \mathcal{F}}{3} r^3 \left(\dot{r} - \sqrt{f + \dot{r}^2}\right)^2. \tag{30}$$

The 'orthogonal' junction conditions for the brane yield the same expression as in (27) and thus the same brane Friedmann equation as before with the important change that the Weyl parameter $\mathcal{C}$ now depends on time. The 'longitudinal' junction conditions can be expressed as

$$\dot{\rho}_b + 3\frac{\dot{r}}{r}(\rho_b + p_b) = -2\mathcal{F}, \tag{31}$$

which differs from the previous conservation law (8) by the nonzero right hand side which represents the energy loss, from the brane point of view, due to the escaping gravitons.

In order to close the system, one must evaluate the rate of graviton production in terms of the brane parameters. In the radiation era, one can show that the energy density loss rate $\mathcal{F}$ is proportional to T^8, so that one can write

$$\mathcal{F} = \frac{\alpha}{12} \kappa^2 \rho^2, \tag{32}$$

where α depends on the number of relativistic degrees of freedom. $\alpha \simeq 0.019$ if all degrees of freedom of the standard model are relativistic.

One can now solve the coupled system consisting of (30), (31) with (32), and the Friedmann equation. The high energy regime is characterized by a rapid growth of the Weyl parameter due to an abundant production of bulk gravitons. In the low energy radiation era, the Weyl parameter approaches a constant value, which means that the production of bulk gravitons becomes negligible.

This asymptotic value for $\mathcal{C}$ can be estimated analytically. If the present description is valid deep enough in the high energy regime, one finds

$$\epsilon_W \equiv \frac{\rho_{Weyl}}{\rho_{rad}} \simeq 2 \times 10^{-3} \tag{33}$$

at the time of nucleosynthesis (with the value of α given above). This result must be compared with the present bound on additional relativistic degrees of freedom allowed during nucleosynthesis, which gives the constraint $\epsilon_W \lesssim 8 \times 10^{-2}$.

5. Bulk scalar field

Although brane cosmology has been mostly studied, out of simplicity, for an *empty* bulk, i.e. with only gravity propagating in the bulk, string models have prompted the analysis of brane cosmology with bulk fields, the simplest example being a bulk scalar field. Such a model can be described by the action

$$\mathcal{S} = \int d^5x \sqrt{-g} \left[\frac{^{(5)}R}{2\kappa^2} - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] + \int_{brane} d^4x \, L_m[\varphi_m, \tilde{h}_{\mu\nu}], \tag{34}$$

where it is assumed that the four-dimensional metric $\tilde{h}_{\mu\nu}$, minimally coupled to the four-dimensional matter fields φ_m in the brane, is conformally related to the induced metric $h_{\mu\nu}$, i.e.

$$\tilde{h}_{\mu\nu} = e^{2\xi(\phi)} h_{\mu\nu}. \tag{35}$$

Variation of the action (34) with respect to the metric yields the five-dimensional Einstein equations (4), where, in addition to the (distributional) brane energy-momentum tensor, there is now the scalar field energy-momentum tensor. Variation of (34) with respect to ϕ yields the equation of motion for the scalar field, which is of the Klein-Gordon type with a distributional source term since the scalar field is coupled to the brane via $\tilde{h}_{\mu\nu}$. This implies that there is another junction condition at the brane location, now involving the scalar field and which is of the form

$$\left[n^A \partial_A \phi \right] = -\xi' T, \tag{36}$$

where $T = -\rho + 3P$ is the trace of the energy-momentum tensor (defined with respect to $h_{\mu\nu}$).

Although the dynamics of the full system is very complicated in general, one can find analytical solutions in some cases, in particular with an exponential potential (Chamblin and Reall, 1999; Langlois and Rodriguez-Martinez, 2001; Charmousis, 2002),

$$V(\phi) = V_0 \exp\left(-\frac{2}{\sqrt{3}}\lambda\kappa\phi\right). \tag{37}$$

For example, there exists a simple class of static solutions, described by the metric

$$ds^2 = -h(R)dT^2 + \frac{R^{2\lambda^2}}{h(R)}dR^2 + R^2 d\vec{x}^2, \tag{38}$$

with

$$h(R) = -\frac{\kappa^2 V_0/6}{1-(\lambda^2/4)}R^2 - \mathcal{C}R^{\lambda^2-2}, \tag{39}$$

where $\mathcal{C}$ is an arbitrary constant, and the scalar field

$$\frac{\kappa}{\sqrt{3}}\phi = \lambda \ln(R). \tag{40}$$

To include a brane in this configuration, one must ensure that the three junction conditions, two for the metric and one for the scalar field, are satisfied. It can be shown that these three junction conditions are equivalent to the following three relations

– a generalized Friedmann equation,

$$H^2 = \frac{\kappa^2}{36}\rho^2 - \frac{h(R)}{R^{2+2\lambda^2}} = \frac{\kappa^2}{36}\rho^2 + \frac{\kappa^2 V_0/6}{1-(\lambda^2/4)}R^{-2\lambda^2} + \mathcal{C}R^{-4-\lambda^2}, \tag{41}$$

– a (non-) conservation equation for the energy density,

$$\dot\rho + 3H(\rho+p) = (1-3w)\xi'\rho\dot\phi, \tag{42}$$

– a constraint on the brane matter equation of state, which must be related to the conformal coupling according to the expression

$$3w - 1 = \frac{\kappa}{\sqrt{3}}\frac{\lambda}{\xi'}. \tag{43}$$

The staticity of the bulk thus allows only a very contrived cosmology in the brane.

In contrast with the empty bulk case, where the required symmetries impose the bulk to be static, one can now find solutions with a non static bulk. For example, with the same potential (37), a solution of the

Einstein/Klein-Gordon bulk equations is given by the metric (Langlois and Rodriguez-Martinez, 2001)

$$ds^2 = \frac{18}{\kappa^2 V_0} e^{4\lambda^2 T}\, e^{4\lambda\sqrt{\lambda^2-1}R}\left(-dT^2+dR^2\right) + e^{4T} d\vec{x}^2, \tag{44}$$

and the scalar field configuration

$$\phi = 4\left(\lambda^2 T + \lambda\sqrt{\lambda^2-1}\, R\right). \tag{45}$$

It is possible to embed a (mirror) symmetric brane in this spacetime, with an equation of state $p_b = w\rho_b$ (w constant). This leads, for a brane observer in the Einstein frame, to a cosmology with a power-law expansion, but once more rather contrived.

References

P. Binétruy, C. Deffayet and D. Langlois, Nucl. Phys. B **565** (2000) 269 [arXiv:hep-th/9905012].

P. Binétruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. B **477** (2000) 285 [arXiv:hep-th/9910219].

H. A. Chamblin and H. S. Reall, Nucl. Phys. B **562**, 133 (1999) [arXiv:hep-th/9903225].

C. Charmousis, Class. Quant. Grav. **19**, 83 (2002) [arXiv:hep-th/0107126].

J. M. Cline, C. Grojean and G. Servant, Phys. Rev. Lett. **83**, 4245 (1999) [arXiv:hep-ph/9906523].

C. Csaki, M. Graesser, C. Kolda and J. Terning, Phys. Lett. B **462**, 34 (1999) [arXiv:hep-ph/9906513]

E. E. Flanagan, S. H. Tye and I. Wasserman, Phys. Rev. D **62**, 044039 (2000) [arXiv:hep-ph/9910498].

J. Garriga and T. Tanaka, Phys. Rev. Lett. **84** (2000) 2778 [arXiv:hep-th/9911055].

A. Hebecker and J. March-Russell, Nucl. Phys. B **608**, 375 (2001) [arXiv:hep-ph/0103214].

P. Horava and E. Witten, Nucl. Phys. B **460**, 506 (1996); Nucl. Phys. B **475**, 94 (1996).

C. D. Hoyle, U. Schmidt, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, D. J. Kapner and H. E. Swanson, Phys. Rev. Lett. **86**, 1418 (2001).

L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690 [arXiv:hep-th/9906064].

P. Kraus, JHEP **9912** (1999) 011 [arXiv:hep-th/9910149].

D. Langlois, K. i. Maeda and D. Wands, Phys. Rev. Lett. **88**, 181301 (2002) [arXiv:gr-qc/0111013].

D. Langlois and M. Rodriguez-Martinez, Phys. Rev. D **64**, 123507 (2001) [arXiv:hep-th/0106245].

D. Langlois, L. Sorbo and M. Rodriguez-Martinez, Phys. Rev. Lett. **89**, 171301 (2002) [arXiv:hep-th/0206146].

T. Shiromizu, K. i. Maeda and M. Sasaki, Phys. Rev. D **62**, 024012 (2000) [arXiv:gr-qc/9910076].